

Scale Dependence of Power Spectrum: Resulting from Non-Bunch-Davies Modes in de Sitter Background

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ABSTRACT: In this paper, we calculate corrections of scalar perturbations spectra resulting from non-Bunch-Davies modes as the general initial states. First, by considering the asymptotic expansion of the Hankel functions up to the higher order of $1/k\eta$, we modify Mukhanov equation and obtain general solutions of it. We nominate this non-trivial excited solution as the fundamental vacuum mode functions during inflation. Then, we calculate power spectrum with this alternative vacuum mode and we obtain modified form of scale-dependent power spectrum with trans-Planckian corrections. Also, unlike the conventional methods, we consider de Sitter space-time instead Minkowski space-time as a background. The method used for renormalization preserves the space-time symmetry, and gives the finite power spectrum and this motivates us to use of excited de Sitter modes. Finally, by considering recent Planck and WMAP results, we give an observational motivation for our non-Bunch-Davies vacuum modes.

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1 Introduction and Motivation

Inflation as the most exciting scenario for cosmologists, naturally solves several puzzles of the standard big bang model [1–3]. The rapid expansion of the universe during inflation is considered to be responsible for homogeneity and isotropy on the large scale structure (LSS) of the universe. On the other hand, the quantum fluctuations of the scalar (inflaton) field are generated from vacuum fluctuations during inflation, and inflation would lead to the growth of the modes of fluctuations in the accelerating phase. By this mechanism, the quantum fluctuations located outside the horizon, freezed and became the classical fluctuations. The physical propagation of scalar curvature is curvature perturbation \mathcal{R} . After these process, the temperature fluctuations $\frac{\Delta T}{T}$ is the observable in the cosmic microwave background (CMB) by Sachs -Wolfe effect and the frozen fluctuations outside the horizon appear in the form of clusters of galaxies in the LSS of the universe. Almost scale invariant power spectrum of primordial fluctuations is the most important prediction of inflation scenario [4–8].

The amazing development of observational cosmology opens new avenues to the very early universe. The scalar spectral index n_s , tensor-to-scalar ratio r [9, 10], the gravitational wave, non-Gaussianity parameter f_{NL} and the isocurvature perturbation in the CMB [11]

are among the most important observational probes. These probes will constrain the initial conditions of inflation model, trans-Planckian physics and alternative models to inflation, etc. One of the most important observations of the recent CMB data from Planck combined with the large angle polarization data from the Wilkinson Microwave Anisotropy Probe (WMAP), gives a strong constraint on scalar spectral index: $n_s = 0.96030.0073$ at 95% CL [12]. These interesting data can be stimulated us to slow-roll inflation for pure de Sitter (dS) space-time or power law inflation (PLI) for non-dS space-time¹. PLI with $a(t) \propto t^p$ and $p > 1$ arises in the context of a canonical scalar field with an exponential potential [13–16]. In [17] proposed a new model of power law inflation for which the scalar spectral index, the tensor-to-scalar ratio and the non-Gaussianity parameter are in excellent agreement with Planck results. However, results of our study confirm a slow-roll PLI form of expansion for the early universe.

On the other hand, recent observational data in the CMB may come from various sources during the cosmic evolution. It is believed that if all the process and transformations during cosmic expansions are linear, thus the temperature fluctuations are Gaussian. Meanwhile, any deviation from linearity in the expansion process or in the transformations between various stages of expansion process, will influence the final observable data [18]. As the oldest non-linear factors influencing the CMB radiation is the non-trivial initial states, that could be an important source to generate scale-dependent power spectrum and non-zero bispectrum. In this paper, we shall propose the generalized non-linear initial modes instead to the linear Bunch-Davies (BD) mode [19] as the non-trivial initial states, and we call them excited dS (ED) modes [20]. We will show that this deviation from BD mode can modify power spectrum and in particular can produce scale-dependent power spectrum fit with recent observations.

Non-trivial initial states have been studied by many authors, including α -vacuum [18], general multi-mode squeezed states [21–26], particle number eigen-states [27], Gaussian and non-Gaussian initial states [28], coherent states and α -states [29, 30], thermal states [31], homogeneous initial states [32] and *calm* states [33].

It is also well known that the inflation can be described in approximate dS space-time [7, 34], thus the study of dS and quantum theory of fields in this background is well motivated. For example, exploring the possible relations between the dS symmetries and the bispectrum of the fluctuations are well studied in [35] in order to set constraints on the initial fluctuations due to the dS symmetry, especially, how scale transformations and special conformal symmetries constrain the correlation functions. The scalar field in dS background is important because most of inflationary models are theorized using the scalar field [36]. It is proved that due to the famous zero-mode problem, quantization of the massless fields in dS can not be done covariantly [37, 38]. For the conventional renormalization procedure in curved space-time [39], vacuum is defined globally while the singularities are removed locally. Thus it vividly breaks the space-time symmetry. This crucial symmetry would be returned in the theory if such divergences are removed by the quantities which are defined globally in

¹For pure dS space-time we have exponentially inflation, for non-dS space-time we have PLI and for almost dS space-time we have slow-roll PLI.

the curved space-time. By using this alternative scheme of renormalization, we will gain an important theoretical motivation for using of ED modes.

Another motivation for considering ED modes as the initial states, arise from the possibility of trans-Planckian effects [40–49]. Today we observe the modes that have physical momenta $k_{phys} = k/a$ which exceed the Planck energy scale Λ at sufficiently far past times. The effects of trans-Planckian corrections to the modes and dynamics of it, can be mimicked by using the modified initial states. We expect our non-linear initial states, i.e. ED modes give the trans-Planckian corrections of power spectrum up to higher order of H/Λ , where H is the Hubble parameter during inflation.

The rest of this paper proceed as follows: in Sec. 2, we first review quantum fluctuations of scalar field during inflation. In this section, we remind equation of motion, field quantization, boundary conditions for vacuum mode selection and in the end we get power spectrum with pure dS mode. In Sec. 3, we propose ED modes instead BD mode and to achieve this important goal we consider asymptotic expansion of the Hankel functions, generalize Mukhanov equation and mode functions and finally we calculate scale-dependent-power spectrum with this excited dS modes in terms of index of Hankel function. Finally, in Sec. 4 and 5 we will present a theoretical and an observational motivations for the ED modes, respectively. For theoretical motivation we modify renormalization method for curved space-time during inflation and for observational motivation we consider recent Planck and WMAP results for inflation. Conclusions will be discussed in Sec. 6.

2 Review of Quantum Fluctuations during Inflation

2.1 Equation of Motion for Scalar Field

The following metric is used to describe the very early universe during inflation:

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 = a(\eta)^2 (d\eta^2 - d\mathbf{x}^2), \quad (2.1)$$

where for the conformal time η the scale factor is defined by $a(\eta)$. There are some models of inflation but the single-field inflation in which a minimally coupled scalar field (inflaton) in dS background, is usually studied in the literatures. The action for single-field models is given by:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[\mathcal{R} - (\nabla\phi)^2 - 2V(\phi) \right], \quad (2.2)$$

where $8\pi G = \hbar = 1$. We use gauge-invariant variables to avoid fictitious gauge modes by introducing comoving curvature perturbation [7],

$$\mathcal{R} = \Psi + \frac{H}{\dot{\phi}} \delta\phi, \quad (2.3)$$

where Ψ is the spatial curvature perturbation. For this gauge-invariant variable one may expand the action (2.2) to second order,

$$S = \frac{1}{2} \int d^4x a^3 \frac{\dot{\phi}^2}{H^2} \left[\dot{\mathcal{R}}^2 - a^{-2} (\partial_i \mathcal{R})^2 \right], \quad (2.4)$$

By defining the Mukhanov variable $v = z\mathcal{R}$ with $z^2 = a^2 \frac{\dot{\phi}^2}{H^2}$, and transitioning to conformal time η leads to the Mukhanov action,

$$S = \frac{1}{2} \int d^3x d\eta \left[(v')^2 + (\partial_i v)^2 - \frac{z''}{z} v^2 \right], \quad (2.5)$$

where the prime is the derivative with respect to conformal time. Also, we define the fourier expansion of the field v :

$$v(\mathbf{x}, \eta) = \frac{d^3k}{(2\pi)^3} \int_k v_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (2.6)$$

By taking variation of the action (2.5), the Mukhanov equation (equation of motion for the mode functions v_k) satisfies [7, 29, 32],

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0. \quad (2.7)$$

2.2 Field Quantization and Boundary Conditions for Vacuum Selection

For quantization of the field, we promote v to quantum operator \hat{v} ,

$$v \rightarrow \hat{v} = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \left(\hat{a}_{\mathbf{k}} v_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger v_k^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right), \quad (2.8)$$

And the Fourier components v_k are expressed via the following decomposition,

$$v_{\mathbf{k}} \rightarrow \hat{v}_{\mathbf{k}} = \frac{1}{\sqrt{2}} (\hat{a}_{\mathbf{k}} v_k(\eta) + \hat{a}_{-\mathbf{k}}^\dagger v_{-k}^*(\eta)). \quad (2.9)$$

The canonical commutation relations for creation and annihilation operators given by:

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}'), \quad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = [\hat{a}_{\mathbf{k}}^\dagger, \hat{a}_{\mathbf{k}'}^\dagger] = 0. \quad (2.10)$$

The *normalization condition* for the quantum mode functions is can establish if and only if we have,

$$v_k^* v_k' - v_k'^* v_k = -i. \quad (2.11)$$

Equation (2.11) provide one of the boundary conditions on the solutions of (2.7) but this doesn't fix the mode functions completely. The second boundary condition that fix the mode functions comes from *vacuum selection*. Real vacuum state for any fluctuations given by [7, 29],

$$\hat{a}_{\mathbf{k}} |0\rangle = 0. \quad (2.12)$$

First, we consider the standard choice, i.e. the Minkowski vacuum in the very early universe, $\eta \rightarrow -\infty$ or $|k\eta| \gg 1$. In this limit, we define the vacuum state by matching our solutions to the Minkowski vacuum mode in the ultraviolet limit, i.e. on small scales when the mode is deep inside the horizon. This constrain fixes the mode functions completely. In this limit, we have $\frac{z''}{z} \rightarrow 0$ and equation (2.7) change to special form as,

$$v_k'' + k^2 v_k = 0. \quad (2.13)$$

For this case, we have a unique solution as,

$$v_k = \frac{e^{-ik\eta}}{\sqrt{k}}. \quad (2.14)$$

After having two boundary conditions (2.11) and (2.14), the suitable mode completely confirmed on the all scales [7].

2.3 Power Spectrum with Pure de Sitter Mode

For exponentially inflation in pure dS space (i.e. $H = \text{constant}$), we have equation of state $\omega = -1$ and $\frac{z''}{z} = \frac{2}{\eta^2}$. By solving equation (2.7) with two boundary conditions (2.11) and (2.14), and with the positive frequency condition in the limit $\eta \rightarrow -\infty$, we have Bunch-Davies mode [7, 27, 32] as,

$$v_k(\eta) = \frac{1}{\sqrt{k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}, \quad (2.15)$$

For calculation of power spectrum, we need to compute the following quantity [7, 29],

$$\langle \hat{v}_k(\eta) \hat{v}_{k'}(\eta) \rangle = \langle 0 | \hat{v}_k(\eta) \hat{v}_{k'}(\eta) | 0 \rangle = \frac{1}{2} (2\pi)^3 \delta^3(k + k') |v_k(\eta)|^2, \quad (2.16)$$

where we use (2.9) for obtain the last equation. Next, we should introduce some standard quantities in terms of curvature perturbation $\mathcal{R}_k(\eta)$,

$$\langle \hat{\mathcal{R}}_k(\eta) \hat{\mathcal{R}}_{k'}(\eta) \rangle = (2\pi)^3 \delta^3(k + k') P_{\mathcal{R}}, \quad (2.17)$$

$$\Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}}, \quad (2.18)$$

where

$$\mathcal{R}_k(\eta) = \frac{v_k(\eta)}{z} = \frac{v_k(\eta)}{a} \left(\frac{H}{\dot{\phi}}\right). \quad (2.19)$$

Which, $P_{\mathcal{R}}$ is the power spectrum and $\Delta_{\mathcal{R}}^2$ is the dimensionless power spectrum [29].

As we know, the scale dependence of the spectra follows from the *time dependence of the Hubble parameter*² and is quantified by tilt [7],

$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2}{d \ln k}. \quad (2.20)$$

By using equations (2.15 -19), in the super-horizon limit ($k\eta \ll 1$), we have scale invariant power spectrum for BD vacuum mode as,

$$n_s = 1, \quad \Delta_{\mathcal{R}}^2 = \frac{H^2}{(2\pi)^2} \left(\frac{H^2}{\dot{\phi}^2}\right). \quad (2.21)$$

Where H is the Hubble constant for this scale invariant power spectrum (i.e. $n_s = 1$) during exponential inflation in pure dS phase. But observations of CMB and LSS tell us conclusively that the power spectrum of the fluctuations produced during inflation is almost scale invariant (i.e. $n_s \approx 1$) [11, 12]. Therefore, we believe that the geometry of the very early universe is almost dS (or excited dS) with time-dependent Hubble parameter. In the next section, we replace non-linear ED modes instead linear BD mode to obtain deviation from scale invariance in terms of μ and slow-roll parameter ϵ .

²Since the excited dS modes introduced by us (in next section) is applicable for the time-dependent Hubble parameter, so it is certainly expected scale dependence of the spectrum resulting from this alternative mode.

3 Excited de Sitter Modes Instead Bunch-Davies Mode

In this section, by consideration the asymptotic expansion of the Hankel function up to the higher order of $1/k\eta$, we obtain the general solutions of Mukhanov equation. We nominate this non-trivial excited solution as the fundamental vacuum mode functions during inflation. Then, we calculate power spectrum with these alternative vacuum modes and obtain modified form of scale-dependent power spectrum.

The general solutions of mode equation (2.7) can be written as [2, 3, 7, 8]:

$$v_k = \frac{\sqrt{\pi\eta}}{2} \left(A_k H_\mu^{(1)}(|k\eta|) + B_k H_\mu^{(2)}(|k\eta|) \right), \quad (3.1)$$

where $H_\mu^{(1,2)}$ are the Hankel functions of the first and second kind, respectively [2, 8]. But since the function z in mode equation (2.7) is a time-dependent parameter and hence depends on the dynamics of the background space-time, so the equation (2.7) will be very hard to solve *exactly*, and usually used several approximate or numerical solutions valid during slow-roll evolution.

3.1 Asymptotic Expansion of the Hankel Functions

In the far past $|k\eta| \gg 1$ in the very early universe, we are authorized to use asymptotic expansions of the Hankel function up to the higher order of $\frac{1}{|k\eta|}$ as follows [7, 50, 51],

$$H_\mu^{(1,2)}(|k\eta|) \rightarrow \sqrt{\frac{2}{\pi|k\eta|}} \left[1 \pm i \frac{4\mu^2 - 1}{8|k\eta|} - \frac{(4\mu^2 - 1)(4\mu^2 - 9)}{2!(8|k\eta|)^2} \pm \dots \right] \times \exp[\pm i(|k\eta| - (\mu + \frac{1}{2})\frac{\pi}{2})], \quad (3.2)$$

Note that, this asymptotic solution, only and only for $\mu = \frac{3}{2}$ reduce to the exact dS mode (2.15), which consists the first (linear) order of $\frac{1}{|k\eta|}$ and for another value of μ , the modes can contain other non-linear terms of order $\frac{1}{|k\eta|}$.

3.2 Mukhanov Equation and General Mode Functions

For the dynamical inflationary background, we have $\frac{z''}{z} \neq 0$ in equation (2.7) and it is a time-dependent value in terms of conformal time η as following [55],

$$\frac{z''}{z} = \frac{2\alpha}{\eta^2}. \quad (3.3)$$

Also in addition to the variable η , the value of $\frac{z''}{z}$ is depended to the Hankel function index μ . With regard to this point, the equation (2.7) change to general form

$$v_k'' + (k^2 - \frac{2\alpha}{\eta^2})v_k = 0. \quad (3.4)$$

Variable α in (3.3 and 25) given by [7, 55],

$$\alpha = \alpha(\mu) = \frac{4\mu^2 - 1}{8}. \quad (3.5)$$

Consequently, according to the general equation of motion (3.4) and the asymptotic expansion (3.2), the general form of mode functions given as follows,

$$v_k^{gen}(\eta, \mu) = A_k \frac{e^{-ik\eta}}{\sqrt{k}} \left(1 - i \frac{\alpha}{k\eta} - \frac{\beta}{k^2\eta^2} - \dots\right) + B_k \frac{e^{ik\eta}}{\sqrt{k}} \left(1 + i \frac{\alpha}{k\eta} - \frac{\beta}{k^2\eta^2} + \dots\right). \quad (3.6)$$

Where $\beta = \alpha(\alpha - 1)/2$. Note that, we consider $|\eta| = -\eta$ for far past. Also, the general mode (3.6) is a function of both variables η and μ . By consideration of two boundary conditions (2.11), (2.14) and mode function (3.6) up to order of $\frac{1}{k^2\eta^2}$, the positive frequency solutions of the mode equation (3.4) given by,

$$v_k^{ED} = \frac{e^{-ik\eta}}{\sqrt{k}} \left(1 - i \frac{\alpha}{k\eta} - \frac{\beta}{k^2\eta^2} - \dots\right). \quad (3.7)$$

We call this latter solutions as *excited dS modes*³. In addition to the above motivations, the following items can be considered as the benefits of these excited modes:

- Corrections obtained from previous conventional methods for power spectrum is typically of the order of 1, 2 or higher [40- 49]. So it is useful for us to extend the modes up to non-linear order of its parameters. This non-linearity of our vacuum modes appears in the both variables of it, i.e. variable μ and conformal time variable η (In this paper we discuss this topic).
- Primordial Non-Gaussianity of the CMB may be come from various non-linear sources during the cosmic evolution. Any non-linear effect in the expansion process or in the initial conditions (vacuum states) may leave non-linear traces in the observable parameters of CMB [18, 52- 54]. So, introducing of the excited vacuum modes can be considered as a non-linear effect in the initial states (This issue will studied in future works).
- Also, compared with previous dS vacuum, these excited modes could be more complete solution of the general wave equations (3.4) for the general curved space-time, Whereas BD mode is a specific solution for a specific curved space-time (i.e. dS space-time).

3.3 Exact and Approximate Solutions

The ED modes (3.7) are leading to the exact solutions for the $\mu = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \dots$ and are leading to the approximate solutions for another values of μ . For the our cosmological proposes, we investigate $\mu = \frac{1}{2}, \frac{3}{2}, \mu \simeq \frac{3}{2} + \epsilon$ with $\epsilon < 1$ and $\mu = \frac{5}{2}$.

- For the $\mu = \frac{1}{2}$, we have from equation (3.3), $\frac{z''}{z} = 0$. So, the mode function (3.7) leads to exact Minkowski mode (2.14).

³We call these mode functions as "excited dS modes", because in our approach, similar to "Background Field Method", we consider dS mode as the background state field and the ED modes (3.7) as the excited state fields in the curved space-time (Sec. IV). Also, it should be noted that taking into account higher order terms in these modes can mean taking into account the interactive field in the theory.

- In the case of $\mu = \frac{3}{2}$, we obtain from equation (3.3), $\frac{z''}{z} = \frac{2}{\eta^2}$, and the general form of the mode functions (3.7) leads to the exact BD mode:

$$v_k^{BD} = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}. \quad (3.8)$$

For this special case, we have exponentially inflation, $a(t) = e^{Ht}$ or $a(\eta) = -\frac{1}{H\eta}$ with $H = \text{constant}$ for very early universe.

- Also, if we consider $\mu = \frac{5}{2}$, we obtain from equation (3.3), $\frac{z''}{z} = \frac{6}{\eta^2}$. So, we have an exact excited solution as,

$$v_k^{ED} = \frac{1}{\sqrt{k}} \left(1 - \frac{3i}{k\eta} - \frac{3}{k^2\eta^2}\right) e^{-ik\eta}, \quad (3.9)$$

Note that this last mode is the exact solution of Mukhanov equation (3.4) and it is a non-linear solution of order $\frac{1}{k^2\eta^2}$.

Inflation starts in approximate dS space-time and basically in this high energy area of very early universe with varying H , finding a proper vacuum mode is difficult. So, we offer the general form of excited solution (3.7) as the fundamental mode during inflation that asymptotically approaches to flat background in $\eta \rightarrow -\infty$. For this fundamental mode the best values of μ which are confirmed with the latest observational data, is $\mu = \frac{3}{2} + \epsilon$, where $0 < \epsilon < 1$. For this range of μ we have power law inflation (PLI) $a(t) \propto t^p$ with $p > 1$ or $a(\eta) \propto |\eta|^{\frac{1}{2}-\mu}$ with $\mu > 3/2$. But for $\epsilon \ll 1$ we have slow-roll PLI [17].

Important point to note is that if we consider vacuum modes depending on the variable μ , BD vacuum mode is exclusively suitable only for exponentially inflation in pure de Sitter phase with $\mu = 3/2$. But for PLI in the excited dS space, it is better and more logical that we make use general ED modes (3.7) instead BD mode (3.8).

3.4 Power Spectrum with Excited de Sitter Modes

For this alternative mode (3.7), we have

$$P_{\mathcal{R}} = \frac{1}{2a^2} \left(\frac{H^2}{\dot{\phi}^2}\right) |v_k^{ED}(\eta)|^2. \quad (3.10)$$

For super-horizon limit $k\eta \ll 1$, we obtain modified power spectrum in the following general form:

$$\begin{aligned} \Delta_{\mathcal{R}}^2 &= \frac{H^2}{(2\pi)^2} \left(\frac{H^2}{\dot{\phi}^2}\right) (1-\epsilon)^2 \left[\alpha + \frac{\beta^2}{k^2\eta^2}\right] \\ &= \frac{H^2}{(2\pi)^2} \left(\frac{H^2}{\dot{\phi}^2}\right) \left(\frac{2}{2\mu-1}\right)^2 \left[\alpha + \frac{\beta^2}{k^2\eta^2}\right]. \end{aligned} \quad (3.11)$$

For the last equation we use the following relation for η [55],

$$\eta = \frac{-1}{aH} \left(\frac{1}{1-\epsilon}\right). \quad (3.12)$$

Here, to obtain the above relation, we assume the slow-roll parameter ϵ is a constant [55]. For special cases of μ , we obtain as follows:

- For the $\mu = \frac{1}{2}$, we have $\alpha = 0$. So, the power spectrum for this case is vanish.
- In the pure dS phase or $\mu = \frac{3}{2}$, we obtain

$$\alpha = 1, \quad \epsilon = \frac{-\dot{H}}{H^2} = 0. \quad (3.13)$$

So, we have BD mode and (3.11) reduced to the standard scale invariant power spectrum (2.21) for this particular case.

- Also, if we consider $\mu = \frac{3}{2} + \frac{1}{p-1}$ [55], we have PLI with power $p = \frac{1}{\epsilon}$ with $\epsilon < 1$ and we consider for this case,

$$\alpha \neq 1, \quad \epsilon = \text{Constant}. \quad (3.14)$$

So, for ED modes (3.7) up to second order of $1/k\eta$, the modified power spectrum (3.11) change to the following result in terms of μ ,

$$\Delta_{\mathcal{R}}^2 = \frac{H^2}{(2\pi)^2} \left(\frac{H^2}{\dot{\phi}^2} \right) \left[\frac{2\mu+1}{2(2\mu-1)} + (2\mu+1)^2 \frac{(4\mu^2-9)^2}{64k^2\eta^2} \right], \quad (3.15)$$

By considering the trans-Planckian effect that appears as a fixed scale $\eta = \eta_0$ [49] and equations (3.11), the equation (3.15) change to,

$$\Delta_{\mathcal{R}}^2 = \frac{H^2}{(2\pi)^2} \left(\frac{H^2}{\dot{\phi}^2} \right) \left[\frac{2\mu+1}{2(2\mu-1)} + \left(\frac{2\mu+1}{2\mu-1} \right)^2 \left(\frac{4\mu^2-9}{4} \right)^2 \left(\frac{H}{\Lambda} \right)^2 \right]. \quad (3.16)$$

Note that for a given k a finite η_0 is chosen in which the physical momentum corresponding to k is given by some fixed scale Λ , where $\eta_0 = -\frac{\Lambda}{Hk}$ has a finite value. The trans-Planckian corrections in (3.16) are of order $(\frac{H}{\Lambda})^2$. In [48] by using of effective field theory the similar correction has been obtained. Also, for the careful analysis of the different order of trans-Planckian corrections can be checked paper [57].

Equations (3.15) and (3.16) indicate that whatever the value of μ is closer to $3/2$ (or $\epsilon \rightarrow 0$), we have slow-roll PLI instead PLI and the deviation from scale invariant spectrum becomes less and less. However in general, our excited modes (3.7) build a scale-dependent power spectrum during PLI. In the last section of this article, according to recent observations, we examine this important result further. As well as because of obtained second order trans-Planckian correction in (3.16), it is possible that ED modes create non-Gaussian effects in CMB similar to effective field theory method [64]. We will investigate this issue in detail in the future works .

4 Renormalized Power Spectrum

In this section, unlike the conventional renormalization methods, we consider dS space-time instead Minkowski space-time as the background space. This viewpoint preserve symmetry of curved space-time, give finite power spectrum and motivate us to departure from the dS vacuum mode to the ED modes.

For any given modes v_k , the two-point function for quantum field $\hat{\mathcal{R}}$ (2.19) in the Hilbert space is defined by:

$$\langle \mathcal{R}^2 \rangle = \frac{1}{(2\pi)^3} \left(\frac{H^2}{\dot{\phi}^2} \right) \int \frac{|v_k|^2}{2a^2} d^3k. \quad (4.1)$$

Then from (4.1) one can write for pure dS mode (3.8) [20]:

$$\langle \mathcal{R}^2 \rangle_{dS} = \frac{1}{(2\pi)^3} \left(\frac{H^2}{\dot{\phi}^2} \right) \int d^3k \left[\frac{1}{2ka^2} + \frac{H^2}{2k^3} \right]. \quad (4.2)$$

The first term in the right hand side is the usual contribution from vacuum fluctuations in Minkowski space-time and is infinite, and can be eliminated after the renormalization [2, 3], then renormalized power spectrum for the scalar field fluctuations is given by as:

$$\Delta_{\mathcal{R}}^2 = \left(\frac{H}{2\pi} \right)^2 \left(\frac{H^2}{\dot{\phi}^2} \right). \quad (4.3)$$

At the next two subsections the power spectrum is renormalized in the curved space-time by new scheme.

4.1 Modifying of Renormalization Method for Curved Space-time

In QFT in flat space-time, the vacuum expectation value of the energy-momentum tensor diverges which by the normal ordering procedure one overcomes this divergence problem. However in curved space-time, following remedy is usually used [39],

$$\langle T_{\mu\nu} \rangle_{ren} = \langle \Omega | T_{\mu\nu} | \Omega \rangle - \langle 0 | T_{\mu\nu} | 0 \rangle, \quad (4.4)$$

where $|\Omega\rangle$ is the vacuum of the theory and $|0\rangle$ stands for the flat vacuum state. We can write the last equation as follows,

$$\langle T_{\mu\nu} \rangle_{ren} = \langle T_{\mu\nu} \rangle_{cur} - \langle T_{\mu\nu} \rangle_{flat}, \quad (4.5)$$

In this renormalization procedure the vacuum is defined globally while the singularities are removed locally. Indeed the background solutions are no longer solutions of the wave equation in curved space-time, thus it vividly breaks the space-time symmetry. But this symmetry would be returned if such divergences are removed by the quantities which are defined globally in the curved space-time. This interpretation of removing infinity resembles Krein space approach, where the renormalization procedure is accomplished by the help of the negative norm solutions of the wave equation [20, 58- 63]. With applying this new idea, we will have,

$$\langle T_{\mu\nu} \rangle_{ren} = \langle T_{\mu\nu} \rangle_{cur} - \langle T_{\mu\nu} \rangle_{dS}. \quad (4.6)$$

Where we use dS space instead flat (Minkowski) space as background space-time. By using this new scheme of renormalization for calculation of power spectrum, we receive three important purposes:

- (1) Theory preserve space-time symmetry.
- (2) We have finite power spectrum without any additional handmade cutoff.

(3) In addition to BD mode, would be needed other excited mode functions for curved space-time which surely can not be BD mode.

Now, according to (4.5) and (4.6) we can write

$$\langle \mathcal{R}^2 \rangle_{ren} = \langle \mathcal{R}^2 \rangle_{cur} - \langle \mathcal{R}^2 \rangle_{BG}. \quad (4.7)$$

Which the subscripts ren, cur and BG in the above relations means renomalized, curved and Background, respectively.

4.2 Calculation of Renormalized Power Spectrum

First we take the BD mode (3.8) as the fundamental mode in curved space-time and calculate the power spectrum with flat space as a background, then we have:

$$v_k^{cur} = \frac{1}{\sqrt{k}}(1 - \frac{i}{k\eta})e^{-ik\eta} \quad , \quad v_k^{BG} = \frac{1}{\sqrt{k}}e^{-ik\eta}, \quad (4.8)$$

and after doing some calculations one finds:

$$\begin{aligned} \langle \mathcal{R}^2 \rangle_{ren} &= \frac{1}{(2\pi)^3} \left(\frac{H^2}{\dot{\phi}^2} \right) \int d^3k \left(\frac{1}{2ka^2} + \frac{H^2}{2k^3} \right) - \frac{1}{(2\pi)^3} \left(\frac{H^2}{\dot{\phi}^2} \right) \int \frac{d^3k}{2ka^2} \\ &= \frac{1}{2\pi^2} \left(\frac{H^2}{\dot{\phi}^2} \right) \int \frac{dk}{k} \left(\frac{H^2}{2} \right). \end{aligned} \quad (4.9)$$

Note that the power spectrum in this case is obtained as [20],

$$(\Delta_{\mathcal{R}}^2)_{ren} = \left(\frac{H}{2\pi} \right)^2 \left(\frac{H^2}{\dot{\phi}^2} \right). \quad (4.10)$$

Which is the same as (4.3). If we take the ED modes (3.7) as the fundamental mode and calculate the spectrum with dS background, then we have:

$$v_k^{cur} = \frac{1}{\sqrt{k}} \left(1 - i \frac{\alpha}{k\eta} - \frac{\beta}{k^2\eta^2} \right) e^{-ik\eta} \quad , \quad v_k^{BG} = \frac{1}{\sqrt{k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}, \quad (4.11)$$

After doing some straightforward algebra, we obtain renormalized power spectrum as the following general form;

$$(\Delta_{\mathcal{R}}^2)_{ren} = \frac{H^2}{(2\pi)^2} \left(\frac{H^2}{\dot{\phi}^2} \right) \left[(\alpha(1 - \epsilon)^2 - 1) + \frac{\beta^2}{k^2\eta^2} \right]. \quad (4.12)$$

5 Non Bunch-Davies Vacuum Modes in the Light of Planck Results

In this section we present resent Planck results for scalar spectral index of inflation [12] as observational motivation for our initial ED modes. The new data for spectral index shows that the μ must be greater than the 3/2 and this important result stimulate us departure from BD mode to the non-BD modes⁴ as follows,

$$v_k^{BD} = \frac{1}{\sqrt{k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta} \quad \rightarrow \quad v_k^{ED} = \frac{1}{\sqrt{k}} \left(1 - \frac{i\alpha}{k\eta} - \frac{\beta}{k^2\eta^2} - \dots \right) e^{-ik\eta}. \quad (5.1)$$

⁴As a similar work in this issue, the reader can be refer to [65].

On the other hand, the solutions of equation (3.3) for this case (i.e. $\mu > 3/2$) leads to PLI, corresponds to

$$a(t) \propto t^p \quad \text{or} \quad a(\eta) \propto |\eta|^{\frac{1}{2}-\mu}. \quad (5.2)$$

For this case, the Hubble parameter is variable and easy to see that from (5.2), we obtain

$$H(t) = \frac{p}{t}. \quad (5.3)$$

In terms of variable μ , we obtain p as follows,

$$p = \frac{(1/2 - \mu)}{(3/2 - \mu)}, \quad (5.4)$$

and the slow-roll parameter for PLI given by

$$\epsilon = \frac{1}{p} = \frac{(3/2 - \mu)}{(1/2 - \mu)}. \quad (5.5)$$

Slow-roll PLI corresponds to $\epsilon \ll 1$ and $\mu = 3/2 + \epsilon/(1 - \epsilon) \simeq 3/2 + \epsilon$ [55]. For this limit one finds,

$$n_s - 1 \simeq \frac{-2}{p} = \frac{2(3 - 2\mu)}{2\mu - 1}. \quad (5.6)$$

Note that if we consider PLI without slow-roll assumption, one gets

$$n_s - 1 = \frac{2}{1 - p} = 3 - 2\mu. \quad (5.7)$$

Planck results in combination with the large angle polarization data from WMAP requires the value of the scalar spectral index n_s to lie in the range $0.945 \leq n_s \leq 0.98$ [12]. This observational constraint, restricts p in the PLI in the following range,

$$38 \lesssim p \lesssim 101, \quad (5.8)$$

and equivalently we obtain for μ and ϵ ,

$$1.51 \leq \mu \leq 1.53, \quad (5.9)$$

$$0.01 \lesssim \epsilon \lesssim 0.03. \quad (5.10)$$

Since the BD vacuum is used just for pure dS space-time, this above range of μ based on the most recent observations, motivate us to deviations from BD mode. Consequently, non-linear ED modes (3.7) proposed in this paper could be a good candidate for the replacement of the standard BD mode.

6 Conclusions

Resent data from Planck and WMAP motivated us to deviation from BD mode. Based in this fact, we have studied the non-BD modes instead BD mode as the general initial states for primordial fluctuations. We have offered ED modes as the non-BD modes by using asymptotic expansion of the Hankel functions in the general solution of the equation

of motion for the field in the curved space-time.

With using this alternative modes we have calculate the power spectrum for primordial scalar field fluctuations. Our result for the power spectrum is scale-dependent and the corrections are up to the higher order of H/Λ , consistent with the other recent works in this issue and for the dS limit leaded to the standard scale invariant power spectrum.

Then, we have presented the modifying renormalization method for curved space-time during inflation. Pursuing this approach, the power spectrum of the inflation was recalculated, and it was shown that the results are similar to the previous works, but in our calculation the infinity does not appear. This inspires some kind of the renormalization, noting that the theory becomes finite itself. On the other hand, the symmetry of curved space-time has been preserved. Finally, we have presented constraints of scalar spectral index of power spectrum and confirmed slow-roll PLI and non-BD vacuum modes are consistent with these observations. In the future work we will study the primordial non-Gaussianity in the CMB resulting from non-linear terms of ED modes.

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References

- [1] A. H. Guth, " The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," Phys. Rev. D 23, 347 (1981).
- [2] A. Linde, " Particles Physics and Inflationary Cosmology," Harwood Academic, Reading (1991).
- [3] A. R. Liddle, " An Introduction to Cosmological Inflation," (1999), [astro-ph/9901124v1].
- [4] A. A. Starobinsky, " Dynamics of Phase Transition in The New Inflationary Universe Scenario and Generation of Perturbations," Phys. Lett. B 117, 175 (1982).
- [5] S. W. Hawking, " The Development of Irregularities in a Single Bubble Inflationary Universe," Phys. Lett. B 115, 295 (1982).
- [6] A. H. Guth and S. Y. Pi, " Fluctuations in the New Inflationary Universe," Phys. Rev. Lett. 49, 1110 (1982).
- [7] D. Baumann, " TASI Lectures on Inflation," TASI 2009, [hep-th/0907.5424].
- [8] V. Mukhanov, " Physical Foundations of Cosmology," Mar., 2001.
- [9] A. A. Starobinsky, " Spectrum of Relict Gravitational Radiation and The Early State of the Universe," JETP Lett. 30 (1979) 682.
- [10] A. A. Starobinsky, " Cosmic Background Anisotropy Induced by Isotropic Flat-Spectrum Gravitational-Wave Perturbations," 11 (1985) 133.
- [11] E. Komatsu et al., " Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," Astrophys. J. Suppl., [astro-ph/0803.0547].

- [12] P. A. R. Ade, et. al., " Planck 2013 Results. XXII. Constraints on Inflation," and " XXIV. Constraints on primordial non-Gaussianity,"[astro-ph/1303.5082].
- [13] L. F. Abbott and M. B. Wise, " Constraints on Generalized Inflationary Cosmologies,"Nucl. Phys. B 244 (1984) 541.
- [14] F. Lucchin and S. Matarrese, " Power-law Inflation," Phys. Rev. D 32 (1985) 1316.
- [15] V. Sahni, " Scalar Field Fluctuations and Infrared Divergent States in Cosmological Models with Power Law Expansion," Class. Quant. Grav. 5 (1988) L113; "Energy Density of Relic Gravity Waves from Inflation," Phys. Rev. D 42 (1990) 453.
- [16] T. Souradeep and V. Sahni, " Density Perturbations, Gravity Waves and the Cosmic Microwave Background," Mod. Phys. Lett. A7 (1992) 3541 [hep-ph/9208217].
- [17] S. Unnikrishnana and V. Sahni, " Resurrecting Power Law Inflation in the Light Of Planck Results," JCAP 10 (2013) 063,[astro-ph/1305.5260].
- [18] W. Xue and B. Chen, " α -Vacuum and Inflationary Bispectrum," Phys. Rev. D 79(2009), [hep-th/0806.4109].
- [19] T. S. Bunch and P. C. W. Davies, Quantum field theory in de Sitter space-Renormalization by point-splitting," Proc. R. Soc. Lond. A 117, 360 (1978).
- [20] M. Mohsenzadeh, M. R. Tanhayi and E. Yusofi, " Power spectrum with auxiliary fields in de Sitter space," Under review in EPJC, [hep-th/1306.6722].
- [21] F. Nitti, M. Porrati, and J.-W. Rombouts, "Naturalness in cosmological initial conditions," Phys.Rev. D72 (2005) 063503, [hep-th/0503247].
- [22] M. Porrati, " Effective field theory approach to cosmological initial conditions: Self-consistency bounds and non-Gaussianities," [hep-th/0409210].
- [23] R. Holman and A. J. Tolley, " Enhanced Non-Gaussianity from Excited Initial States," JCAP 0805 (2008) 001, [hep-th/0710.1302].
- [24] P. D. Meerburg, J. P. van der Schaar, and P. S. Corasaniti, " Signatures of Initial State Modifications on Bispectrum Statistics," JCAP 0905 (2009) 018, [hep-th/0901.4044].
- [25] J. Ganc, " Calculating the local-type f_{NL} for slow-roll inflation with a non-vacuum initial state," Phys.Rev. D84 (2011) 063514, [astro-ph/1104.0244].
- [26] I. Agullo and S. Shandera, " Large non-Gaussian Halo Bias from Single Field Inflation," JCAP 1209 (2012) 007, [astro-ph/1204.4409].
- [27] I. Agullo and L. Parker, " Non-gaussianities and the Stimulated creation of quanta in the inflationary universe," Phys.Rev. D83 (2011) 063526, [astro-ph/1010.5766].
- [28] N. Agarwal, R. Holman, A. J. Tolley, and J. Lin, " Effective field theory and non-Gaussianity from general inflationary states," JHEP 1305 (2013) 085, [hep-th/1212.1172].
- [29] S. Kundu, "Inflation with General Initial Conditions for Scalar Perturbations," JCAP 1202 (2012) 005, [astro-ph/1110.4688].
- [30] S. Kundu, " Non-Gaussianity Consistency Relations, Initial States and Back-reaction," [astro-ph/1311.1575].
- [31] S. Das and S. Mohanty, " Non-Gaussianity as a signature of thermal initial condition of inflation," Phys.Rev. D80 (2009) 123537, [astro-ph/0908.2305].

- [32] S. Bahrami, E. E. Flanagan, " Primordial non-Gaussianities in single field inflationary models with non-trivial initial states,"[astro-ph/1310.4482].
- [33] A. Ashoorioon and G. Shiu, " A Note on Calm Excited States of Inflation," JCAP 1103 (2011)025, [arXiv:1012.3392].
- [34] A. D. Linde, "Inflationary Cosmology," Lect. Notes Phys. 738 (2008), [hep-th/0705.0164].
- [35] J. M. Maldacena and G. L. Pimentel, " On graviton non-Gaussianities during inflation," JHEP 1109:045,2011[hep-th/1104.2846].
- [36] C. Wetterich, " Cosmology and the fate of dilatation symmetry," Nucl. Phys., B 302: 668, (1988).
- [37] B. Allen, " Vacuum states in de Sitter space," Phys. Rev. D 32,3136 (1985).
- [38] B. Allen and A. Folacci," Massless minimally coupled scalar field in de Sitter space," Phys. Rev. D 35, 3771 (1987).
- [39] N. D. Birrel and P. C. W. Davies, " Quantum Fields in Curved Space," Cambridge University Press," Cambridge (1982).
- [40] K. Goldstein and D. A. Lowe, " Initial state effects on the cosmic microwave background and transPlanckian physics," Phys.Rev. D67 (2003) 063502, [hep-th/0208167].
- [41] J. Martin and R. H. Brandenberger, " The TransPlanckian problem of inflationary cosmology," Phys.Rev. D63 (2001) 123501, [hep-th/0005209].
- [42] H. Collins and R. Holman, " Trans-Planckian enhancements of the primordial non-Gaussianities," Phys.Rev. D80 (2009) 043524, [arXiv:0905.4925].
- [43] R. Easther, B. R. Greene, W. H. Kinney, and G. Shiu, " Inflation as a probe of short distance physics," Phys.Rev. D64 (2001) 103502, [hep-th/0104102].
- [44] R. Brandenberger and P. M. Ho, " Noncommutative space-time, stringy space-time uncertainty principle, and density fluctuations," Phys.Rev. D66 (2002) 023517, [hep-th/0203119].
- [45] F. Lizzi, G. Mangano, G. Miele, and M. Peloso, " Cosmological perturbations and short distance physics from noncommutative geometry," JHEP 0206 (2002) 049, [hep-th/0203099].
- [46] R. Easther, B. R. Greene, W. H. Kinney, and G. Shiu, "A Generic estimate of transPlanckian modifications to the primordial power spectrum in inflation," Phys.Rev. D66 (2002) 023518, [hep-th/0204129].
- [47] U. H. Danielsson, " Inflation, holography, and the choice of vacuum in de Sitter space," JHEP 0207 (2002) 040, [hep-th/0205227].
- [48] N. Kaloper, M. Kleban, A. E. Lawrence, and S. Shenker, " Signatures of short distance physics in the cosmic microwave background," Phys.Rev. D66 (2002) 123510, [hep-th/0201158].
- [49] U. H. Danielsson, " A note on inflation and transplanckian physics," Phys.Rev. D66 (2002) 023511, [hep-th/0203198].
- [50] G. B. Arfken, et al. " Matematical Method for Physicists," Academic Press, Inc., 1985; M. Abramowitz and I. Stegun, "Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables," Dover (1974).

- [51] C. Armendariz-Picon and E. A. Lim, " Vacuum Choices and the Predictions of Inflation," JCAP 0312:006,2003, [hep-th/0303103].
- [52] H. A. Buchdahl, "Non-linear Lagrangians and cosmological theory," Mont. Not. R. Astr. Soc. 150, (1970).
- [53] A. Peacock and S. J. Dodds, " Non-linear evolution of cosmological power spectra," Mon. Not. R. Astron. Soc. 280,(1996).
- [54] L. Verde1, L. Wang, A. F. Heavens and M. Kamionkowski, " Large-scale structure, the cosmic microwave background and primordial non-Gaussianity," Mon. Not. R. Astron. Soc. 313,(2000).
- [55] E. D. Stewart, D. H. Lyth, " A more accurate analytic calculation of the spectrum of cosmological perturbations produced during inflation," Phys. Lett. B (1993) [gr-qc/9302019].
- [56] A. Kempf and J. C. Niemeyer, " Perturbation spectrum in inflation with cutoff," Phys. Rev. D 64, 103501 (2001),[astro-ph/0103225].
- [57] J. Martin and R. H. Brandenberger, " On the Dependence of the Spectra of Fluctuations in Inflationary Cosmology on Trans-Planckian Physics," Phys.Rev. D68 (2003) 063513, [hep-th/0305161];
R. Brandenberger and J. Martin, " Trans-Planckian Issues for Inflationary Cosmology," [astro-ph/1211.6753].
- [58] J. P. Gazeau, J. Renaud and M. V. Takook, " Gupta-Bleuler quantization for minimally coupled scalar fields in de Sitter space," Class. Quant. Grav. 17, 1415 (2000), [gr-qc/9904023].
- [59] M. V. Takook and S. Rouhani,"Quantum Gravity in Krein Space Quantization," [gr-qc/1208.5562v1].
- [60] M. V. Takook, " Negative Norm States in de Sitter Space and QFT without Renormalization Procedure," Int. J. Mod. Phys. E 11, 509 (2002), [gr-qc/0006019];
S. Rouhani and M. V. Takook, "Tree-level Scattering Amplitude in de Sitter Space," Euro. Phys. Lett. 68, 15 (2004), [gr-qc/0409120].
H. Pejhan, M. V. Takook and M. R. Tanhayi, " Casimir Effect For a Scalar Field via Krein Quantization," Annals of Physics 341 (2014),[math-ph/1204.6001].
- [61] A. Refaei, M. V. Takook, " Scalar effective action in Krein space quantization," Phys. Lett. B 704, 326 (2011), [gr-qc/1109.2693].
- [62] M. Mohsenzadeh, A. Sojasi and E. Yusofi,"Spectrum of Gravitational Waves in Krein Space Quantization," Mod. Phys. Lett. A 26, 2697 (2011),[gr-qc/1202.4975];
M. Mohsenzadeh, S. Rouhani and M. V. Takook, "Power Spectrum in Krein Space Quantization," Int. J. Theor. Phys. (2009) 48, [gr-qc/0811.0982].
- [63] B. Forghan, M. V. Takook and A. Zarei, " Krein Regularization of QED," Annals of Physics 327(2012), [hep-ph/1206.2796].
- [64] L. Senatore, " TASI 2012 Lectures on Inflation," Published by World Scientific Publishing Co. Pte. Ltd., 2013.
- [65] X.Chen and Y. Wang, " Non-Bunch-Davies Anisotropy," [hep-th/1306.0609].